

MULTIPLE CHOICE. Consider the DEs

SCORE: ____ / 3 PTS

[1] $(5r + 1) \frac{dr}{d\theta} = 2\theta + 1$

[2] $x''y - y^2 = 2x$

[3] $(5w + 1)du + (\ln w - 3u)dw = 0$

(where w is the independent variable)

Which of the DE above are linear ? Circle the correct answer below.

(a) none are linear

(b) only [1] is linear

(c) only [2] is linear

(d) only [3] is linear

(e) only [1] & [2] are linear

(f) only [1] & [3] are linear

(g) only [2] & [3] are linear

(h) all are linear

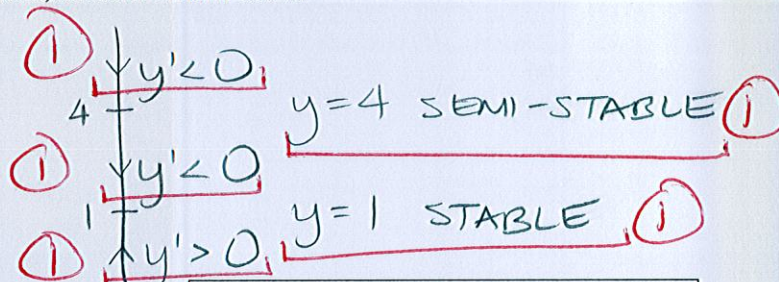
3

Consider the autonomous DE $y' = (y - 4)^2(1 - y)$.

SCORE: ____ / 7 PTS

- [a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.

$$(y - 4)^2(1 - y) = 0 \rightarrow y = 1, 4$$



- [b] If $y = f(x)$ is a solution of the DE such that $f(2) = 5$, what is $\lim_{x \rightarrow \infty} f(x)$? [HINT: Sketch a possible graph of $y = f(x)$.]

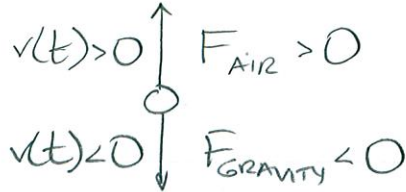
4 ①

- [c] If $y = g(x)$ is a solution of the DE such that $g(5) = -3$, what is $\lim_{x \rightarrow \infty} g(x)$?

1 ①

Write a differential equation for the velocity $v(t)$ of a falling object if the air resistance is proportional to the square of the velocity. Assume that $v(t) > 0$ corresponds to the object moving upward, $v(t) < 0$ corresponds to the object moving downward. (NOTE: This is NOT the same problem as in the homework.)

SCORE: ____ / 3 PTS



$$m \frac{dv}{dt} = -mg + kv^2$$

Below the equation, three red brackets are drawn under the terms $m \frac{dv}{dt}$, $-mg$, and kv^2 respectively. Under each bracket is a circled number 1.

MUST HAVE
CORRECT SIGN

FILL IN THE BLANKS.

SCORE: _____ / 3 PTS

[a] The order of the DE $y^{10} - y^{(7)}y^4 = (x^5 + y''')^6$ is 7 1

[b] If $y = \sqrt{x+9}$ is a solution of the DE $y'' = f(x, y, y')$, the largest possible interval of definition is $(-9, \infty)$

$$y' = \frac{1}{2}(x+9)^{-\frac{1}{2}} \quad y'' = -\frac{1}{4}(x+9)^{-\frac{3}{2}}$$

NOTE: -9 IS NOT
IN THE INTERVAL

Consider the IVP $y' = 5x - 10y$, $y(1) = -2$.

SCORE: ____ / 5 PTS

Use Euler's method with $h = 0.2$ to estimate $y(1.4)$.

$$y(1.2) \approx y(1) + y'(1)(0.2) = -2 + (5(1) - 10(-2))(0.2)$$

$$\overset{\textcircled{2}}{-2} + \underbrace{25(0.2)} = \underbrace{3}_{\textcircled{1}}$$

$$y(1.4) \approx y(1.2) + y'(1.2)(0.2) \approx \underbrace{3}_{\textcircled{1}} + \underbrace{(-24)(0.2)}_{\textcircled{1}} = \underbrace{-1.8}_{\textcircled{1}}$$

What does the Existence & Uniqueness Theorem tell you about the IVP $(\sin x)y' - y^{\frac{5}{2}} = 0$, $y(\frac{\pi}{4}) = 0$?

SCORE: ____ / 3 PTS

Justify your answer properly, but briefly.

$\textcircled{\frac{1}{2}}$ $y' = \frac{y^{\frac{5}{2}}}{\sin x} = f$, so $f_y = \frac{\frac{5}{2}y^{\frac{3}{2}}}{\sin x}$

WHICH IS NOT DEFINED / CONTINUOUS
AROUND $(\frac{\pi}{4}, 0)$ WHERE $y < 0$

SO $\textcircled{\frac{1}{2}}$ E+U TELLS US NOTHING $\textcircled{1}$

Consider the DE $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^2$.

SCORE: ____ / 6 PTS

[a] Is $y = Ax^3 + x^2 + Bx$ a family of solutions of the DE ?

$$y' = 3Ax^2 + 2x + B$$

$$y'' = 6Ax + 2$$

$$\begin{aligned} x^2 y'' - 2x y' + 2y &= \overset{\textcircled{1}}{x^2(6Ax+2)} - \overset{\textcircled{1}}{2x(3Ax^2+2x+B)} + \overset{\textcircled{1}}{2(Ax^3+x^2+Bx)} \\ &= \overset{\textcircled{1}}{2Ax^3} \overset{\textcircled{1\frac{1}{2}}}{\textcircled{\frac{1}{2}}} \end{aligned}$$

NO $\textcircled{1\frac{1}{2}}$

[b] If the answer to [a] is "YES", solve the IVP consisting of the DE and the initial conditions $y(1) = -1$, $y'(1) = 3$.
If the answer to [a] is "NO", skip this part.